Abstract—In this abstract, we present an overview of recent results on dynamic walking and whole-body motion planning for humanoid robots. First, we present the field of randomized algorithms used for constrained motion planning, and their application to humanoid whole-body motion planning. Further on, we introduce humanoid small-space controllability, a theoretical property relying on dynamic walking. Such a property leads to a sound method that extends whole-body motion planning algorithms to whole-body and walk planning. We illustrate this method by use examples on the HRP-2 platform.

I. INTRODUCTION

During the last twenty years, impressive progress has been achieved in humanoid robot hardware and control. This leads to a rising need for software and algorithms improving the usability and autonomy of those robots. One important area of research focuses on the development of robust and general motion generation techniques for safe and autonomous operation in human environments, such as offices or homes.

Motion planning for humanoid robots is challenging for several reasons. First, the computational complexity of classic motion planning algorithms is exponential in the number of Degrees of Freedom (DoFs) of the considered system, which is high for humanoid cinematic trees. Second, a humanoid robot is an under-actuated system: the DoFs that control the position and orientation of the whole robot in space are not directly controlled, they derive from the articulation DoFs of the robot legs. Those latter should be controlled with care to guarantee dynamically balanced motions, for manipulation or navigation.

There are two main ways of using motion planners to generate dynamically balanced robotic motions. The more general one is to plan in a robot dynamic space. By taking into account both robot configuration and velocity, motions that satisfy dynamic balance constraints can be generated at a planning phase. When planning motion for humanoid robots, this is a particularly costly approach, as the size of the space to explore is augmented with the robot velocity and footprint positions. The other way is to first plan a geometric path that can be approximated by a dynamic trajectory in a second step. The approach we present here falls into the second category.

The planning algorithm we present here considers exact models of a humanoid robot and its environment. It is used to solve navigation and manipulation problems. Our planner is a two-step algorithm: a first collision-free path is computed in the space of quasi-statically balanced configurations, then this first path is approximated by a sequence of dynamic walking trajectories. The proof of correctness of the algorithm is based on the concept of small-space controllability. This property allows, under some assumptions, to approximate any non necessarily admissible path, by a sequence of admissible trajectories. In our context we prove that dynamic walking makes humanoid robots small-space controllable. Our planner is designed for perfectly modeled indoor environments, where the floor is horizontal and flat.

II. WHOLE-BODY MOTION PLANNING

When planning a whole-body motion for a humanoid robot, one difficult challenge is to cope with the curse of dimensionality. The complexity of motion planning is exponential in the dimension of the configuration space (C) to explore. When dealing with high-dimensional configuration spaces, it is typically impossible to explicitly represent them, leading to the use of randomized sampling techniques to solve global planning problems. In the past fifteen years, Probabilistic Roadmaps [2] and Rapidly exploring Random Trees (RRT) [3] have been developed and used to solve many high-dimensional planning problems, see [4] and [5] for comprehensive overviews. When using sampling techniques on a humanoid robot, another difficulty is to take into account balance constraints, i.e. to generate random configurations on zero volume submanifolds of C. This problem has been investigated with success during the last few years, [6] presents an exhaustive survey of Jacobian-based methods. Other recent contributions [7] present sophisticated constrained motion planning techniques based on higher-dimensional continuation. Let us present in more details a simple adaptation of the RRT algorithm to constrained motion planning, that was first introduced in [8].

A. Constraint Solver

The purpose of the constraint solver is to find the root $q$ of a non-linear $C^1$ function $f(q)$ with a tolerance of $\epsilon$. Algorithm 1 implements a Newton-Raphson method: starting from an initial value of $q$, $q$ is updated iteratively by $-\alpha \frac{\partial f}{\partial q}(q)^+ f(q)$, where $\alpha$ denotes a gain and $\frac{\partial f}{\partial q}(q)^+$ denotes the Moore-Penrose pseudo-inverse of the Jacobian of $f(q)$. 

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Algorithm 1: SolveConstraints(q, f, ϵ): find q such that f(q) = 0

\( i = 0 \)

while \( \|f(q)\| > \epsilon \) and \( i \leq i_{\text{max}} \) do

// \( (.)^\dagger \) denotes the Moore-Penrose pseudo-inverse

\( q \leftarrow q - \alpha \left( \frac{df}{dq}(q) \right)^\dagger f(q) \)

\( i \leftarrow i + 1 \)

// Make \( \alpha \) tend toward \( \alpha_{\text{max}} \)

\( \alpha \rightarrow \alpha_{\text{max}} - \omega(\alpha_{\text{max}} - \alpha) \)

end while

if \( \|f(q)\| \leq \epsilon \) then

return q

else

return failure

end if

B. Goal Manifold Sampling

The way we generate a goal configuration is the following:

1) Shoot a random configuration \( q_{\text{rand}} \) in \( C \) with uniform distribution.

2) Call SolveConstraints (Algorithm 1) on \( q_{\text{rand}} \), with \( f(q) \) defined by the intersection of the planning and goal manifolds \( M \cap M_g \).

3) If success, check for collisions.

Fig. 1 shows resulting random configurations which satisfy both balance \( (M) \) and reaching \( (M_g) \) constraints for the HRP-2 robot.

Fig. 1. Random goal configurations solving a reaching task. All the configurations are balanced and collision-free, and the right hand of the character reaches the orange ball.

C. Random Extensions on a Constrained Manifold

Tree-based randomized motion planners, such as RRT, explore configuration spaces by iterating random extensions. The equivalent of a random extension on a constrained manifold \( M \), defined by the constraint function \( f \), starts from a valid configuration \( q_{\text{near}} \in M \), and extends the tree towards a random configuration \( q_{\text{rand}} \), while keeping the constraints satisfied. Fig. 2 illustrates such an extension. [9] presents a formal proof that projection-based constrained random motion planning on a fixed dimension manifold is probabilistically complete.

III. SMALL-SPACE CONTROLLABILITY: FROM STATICALLY BALANCED PATHS TO DYNAMIC WALK TRAJECTORIES

This work presents a constructive proof that statically balanced, collision-free path for a legged robot sliding on the ground can be approximated by a dynamically balanced, collision-free walk trajectory. The proof is based on small-space controllability and a preliminary version of it was presented in [10].

A. Small-Space Controllability

A robotic system is controllable if for any two configurations \( q_1 \) and \( q_2 \), there exists a trajectory going from \( q_1 \) to \( q_2 \). It is small-space controllable if for all configurations \( q \), for all \( \epsilon > 0 \), there exists \( \eta > 0 \) such that all the configurations contained in the ball of center \( q \) and radius \( \eta \) are reachable by trajectories included in the ball of center \( q \) and radius \( \epsilon \).

Fig. 3 shows an illustration of this property.

Fig. 3. The small-space controllability local property: any configuration \( q' \) at a distance less than \( \eta \) is reachable from \( q \) by an admissible trajectory included in a ball of size \( \epsilon \).

Theorem 1: Any collision-free path of a small-space controllable system can be approximated by a sequence of both collision-free and admissible trajectories. Thus, small-space controllability reduces trajectory planning problems to geometric path planning problems.

B. Dynamic Walking Makes Humanoid Robots Small-Space Controllable

Theorem 2: A quasi-statically walking robot is not small-space controllable. A dynamically walking robot is.

Proof: The proof of this theorem is based on the fact that the criterion for dynamic balance (ZMP) depends on a robot Center of Mass (CoM) position, and on its second derivative. By making the CoM move fast enough, it is possible to achieve dynamic walking while keeping the CoM in an arbitrary small neighborhood. Fig. 4 illustrates this property.
IV. APPLICATION TO MOTION PLANNING

Given a statically balanced sliding path \( p \) generated by any constrained motion planner, we start by placing footprints corresponding to the nominal walk pattern of the robot. When animating the path into a dynamic walk trajectory, collisions may appear between the robot and its environment. If so, it is necessary to approximate more closely \( p \) by a walk trajectory. To do so, we use the small-space controllability property of the system: we accelerate the steps and make them smaller. The walk trajectory corresponding to smaller and faster steps converges toward the sliding path, which guarantees the algorithm soundness.

Experimental results

In the problem shown in Fig. 5 the robot has to put a ball on a shelf, in a constrained apartment environment. The final configuration is defined implicitly as a desired hand position. We have applied our planner to generate a whole-body walking motion that solves the hand reaching task. The solution sliding path is constrained between the table on the right and the lamp on the left. This passage is too narrow for the robot nominal walk parameters. When executing the walk motion resulting from our algorithm, the robot left hand is only a few centimeters away from the lamp.

V. CONCLUSION

We have presented a simple algorithm for constrained motion planning and used it within a novel, well-grounded strategy for humanoid whole-body manipulation planning including locomotion. The locomotion algorithm is based on a formal small-space controllability property of humanoid robots. An important point is that this strategy only holds for dynamic walking robots, and not for quasi-static walking ones. We have used our motion planner on challenging examples, and validated the generated motions on a real platform.

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REFERENCES